Slide 1

Numbers, Numbers, & More Numbers

Making sense of all the numbers

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Slide 2

UNITS! UNITS! UNITS!

- Joe's 1st rule of Physical Sciences - watch the units.
- The ability to convert units is fundamental, and a useful way to solve many simple problems.
- Units also provide the context for numbers.

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Slide 3

11

- Good number at the craps table.
- Bad number for an IQ.
- Okay number for a shoe size.

They are all "elevens" but they are each very different things.

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Slide 4

UNITS! UNITS! UNITS!

Numbers have no meaning without UNITS! UNITS! UNITS!

The unit provides the context to the number.

A number is just a number, but a number with an appropriate unit is a datum (singular of data) - a piece of information.

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Slide 5

Data

11 pounds

11 dollars

11 points

These are better than just “elevens”, these are data, the 11 has some context – but it could have more!

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Slide 6

Data

11 pounds of raisins vs. 11 pound baby vs. 11 pounds of sand

Our units are now even more specific, providing even greater context to the number, allowing better analysis of the meaning of the number.

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Chemical Units

- SI units - Systems Internationale - these are the standard units of the physical sciences (sometimes called the metric system).
- Units are chosen to represent measurable physical properties.
- Two types of units: “Pure” and “Derived”.

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Pure Units

Represent indivisible physical quantities:

Mass – expressed in “kilograms” (kg)
Length – expressed in “meters” (m)
Time – expressed in “seconds” (s)
Charge – expressed in “Coulombs” (C)

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Derived Units

Derived units are combinations of pure units that represent combinations of properties:

Speed – meters/second (m/s) – a combination of distance and time

Volume – m^3 – combination of the length of each of 3 dimensions

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SI units

The official standard units are all metric units. The nice thing about the standard system is that the units are all self-consistent: when you perform a calculation, if you use the standard unit for all of the variables, you will get a standard unit for the answer without having to expressly determine the cancellation of the units.

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It's all about the DATA folks!

The goal in any experimental science is to use measurement and observation as arguments in support of a thesis.

Data is NOT an end unto itself.

Data is part of a narrative. To be a good scientist, you need to learn to use data to craft an argument.

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"Data" has a lot of subtlety

"four"
"4"
"4.0"
"4.00"
"4.00 pounds"
"4.00 pounds of carbon"
"4.00 pounds of carbon in the brain of Tyrannosaurus"

In our everyday speak, we use these interchangeably. But they aren't!

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The UNITS! UNITS! UNITS! mean everything

"4.00"
"4.00 pounds"
"4.00 pounds of carbon"
"4.00 pounds of carbon in the brain of Tyrannosaurus"
"4.00 pounds of carbon in the brain of Teddy the Tyrannosaur whom I bred in my basement"

These are not the same thing. "4.00" could be anything: 4 dollars in my pockets, 4 toes on my left foot, 4 ex-wives…
Specificity is important – it avoids ambiguity!

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"Data" has a lot of subtlety

"four pounds of carbon in the brain of Tyrannosaurus"
"4 pounds of carbon in the brain of Tyrannosaurus"
"4.0 pounds of carbon in the brain of Tyrannosaurus "
"4.00 pounds of carbon in the brain of Tyrannosaurus "

Beyond the UNITS! UNITS! UNITS!, the numbers themselves include information.

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4 is not 4.0 is not 4.00 is not 4.0000

A mathematician wouldn't make a distinction.
Your grandma wouldn't make a distinction – unless she's a scientist.
A scientist makes a SIGNIFICANT distinction.

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Significant Figures

- Units represent measurable quantities.
- Units contain information.
- There are limits on the accuracy of any piece of information.
- When writing a “data”, the number should contain information about the accuracy

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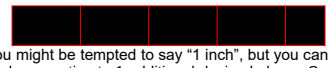
Sig Figs

Suppose I measure the length of my desk using a ruler that is graduated in inches with no smaller divisions – what is the limit on my accuracy?



You might be tempted to say “1 inch”, but you can always estimate 1 additional decimal place. So the answer is 0.1 inches.

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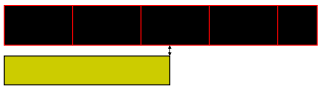
Sig Figs



The green block is about 40% of the way from 2 to 3, so it measures 2.4 inches!

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Accuracy

So, the green block is 2.4 inches long. This is 2 "significant digits" – each of them is accurately known.

Another way of writing this is that the green block is 2.4 +/-0.05 inches long meaning that I know the block is not 2.3 in and not 2.5 in, but it could be 2.35 or 2.45 inches (both would be rounded to 2.4 inches).

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Sig Figs

2.4 inches must always be written as 2.4 inches if it is data.

2.40 inches = 2.400 inches = 2.4 inches BUT NOT FOR DATA!

The number of digits written represent the number of digits measured and KNOWN!

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Ambiguity

Suppose I told you I weigh 200 pounds. How many sig figs is that?

It is ambiguous – we need the zeroes to mark positions relative to the decimal place. Even if that measurement is 200 +/- 50 pounds, I can't leave the zeroes out!

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Scientific Notation

To avoid this ambiguity, numbers are usually written in scientific notation.

Scientific notation writes every number as
#.#### multiplied by some space marker.

For example 2.0×10^2 pounds would represent my weight to TWO sig figs.

The $10^{\#}$ markes the position, so I don't need any extra zeroes lying around.

200 2.00

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Examples of Scientific Notation

$0.00038340\ g = 3.8340 \times 10^{-4}\ g$

- trailing zeroes after decimal are always significant. Leading zeroes are never significant

$200\ lbs = 2 \times 10^2\ lbs = 2.0 \times 10^2\ lbs = 2.00 \times 10^2\ lbs$

- place markers are ambiguous

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Scientific Notation

Only sig figs are written. All digits that are written are significant.

1.200×10^4 – 4 sig figs

1.0205×10^{-1} – 5 sig figs

No ambiguity ever remains!

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How many significant figures are there in the number 0.006410?

Preceding zeroes are NEVER significant.

Trailing zeroes are significant IF YOU DON'T NEED THEM.

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SI units and Latin prefixes

Sometimes, SI units are written with a prefix indicating a different order of magnitude for the unit.

For example, length should always be measured in meters, but sometimes (for a planet) a meter is too small and sometimes (for a human cell) a meter is too large

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Latin Prefixes

M = Mega = 1,000,000 = 10^6
k = kilo = 1,000 = 10^3
c = centi = 1/100 = 10^{-2}
m = milli = 1/1000 = 10^{-3}
μ = micro = 1/1,000,000 = 10^{-6}

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To date

1. Accuracy
   1. Sig figs tell you how well you know the value of something
   2. Scientific notation allows you to express it unambiguously.

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Slide 29

Units! Units! Units!
What is it?
length, volume, weight, energy, charge…

How big is it?
inches? Feet? Yards? Miles? Parsecs?
nm, cm, m, km, Mm, Gm

What else could it be?
It's a foot long, what does it weigh?
It's a gallon big, what does it weigh?
Etc.

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Prefixes & Units

So, if I measure a planet and determine it to be 167,535 meters in circumference, this can be written a number of ways.

167535 m
1.67535×10^5 m
167.535×10^3 m = 167.535 km

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Other systems

The metric system isn't the only system of measurement units. Any arbitrary system of units could be used, as long as the specific nature of each unit and its relationship to the physical property measured was defined.

The "English units" we use in the USA is an example of another system of units.

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Converting Between Systems

If two different units both apply to the same physically measurable property – there must exist a conversion between them.

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Converting Between Systems

If I am measuring length in "Joes" and Sandy is measuring length in "feet" and Johnny is measuring length in "meters", since they are all lengths there must exist a reference between them.

I measure a stick and find it to be 3.6 "Joes" long. Sandy measures it and finds it to be 1 foot long, while Johnny measures it and finds it to be 0.3048 meters long.

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Conversion factors

That means:

1 ft = 3.6 Joes
1 ft = 0.3048 m

This would apply to any measurement of any object

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Dimensional Analysis

Also called the “Factor-label Method”.

Relies on the existence of conversion factors.

By simply converting units, it is possible to solve many simple and even mildly complex problems.

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UNITS! UNITS! UNITS!

It’s always all about the units!

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Slide 37

Conversion Factors

IT IS THE POWER OF ONE!

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Conversion Factors

Dimensional analysis treats all numerical relationships as conversion factors of 1, since you can multiply any number by 1 without changing its value.

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1 foot = 12 inches

This is really two different conversion factors – two different "ones"

$$\frac{1\ foot}{12\ inches} = 1$$

$$\frac{12\ inches}{1\ foot} = 1$$

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One is Most Powerful

"One" is the multiplicative identity – you can multiply any number in the universe by 1 without changing its value.

Multiplying by 1 in the form of a ratio of numbers with units will NOT change its value but it WILL change its units!

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The simplest Example

I am 73 inches tall, how many feet is that?

- I know you can do this in like 10 seconds, but HOW do you do it?

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The Path

The first thing you need to ask yourself in any problem is….?

What do I know?

The second thing you need to ask yourself in any problem is…?

What do I want to know? (Or, what do I want to find out?)

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The Path

The solution in any problem is a question of finding the path from what you know to what you want to know.

In a dimensional analysis problem, that means finding the conversion factors that lead from what you know to what you want to know.

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Slide 44

The simplest Example

I am 73 inches tall, how many feet is that?

The thing I know - START

$$73\ inches\ \frac{???}{???} = ?\ feet$$

The thing I want - FINISH

THE PATH – how I get from START to FINISH

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The Path

$$73\ inches\ \frac{???}{???} = ?\ feet$$

The first step in ANY problem is always half-obvious!

Whatever I'm going to do next, I need to get rid of "inches". [I don't want it, it needs to change!]

$$73\ \text{inches}\ \frac{???}{?\ \text{inches}} = ?\ \text{feet}$$

The path can have 1 step or a thousand steps. The 1 step solution is always obvious (although you may not know it). I change the unit I have into the unit I want.

$$73\ \text{inches}\ \frac{?\ \text{feet}}{?\ \text{inches}} = ?\ \text{feet}$$

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The Path

$$73 \text{ inches } \frac{?\,\text{feet}}{?\ \text{inches}} = ?\,\text{feet}$$

In this case, I know the conversion: 1 foot = 12 inches

$$73 \text{ inches } \frac{1\text{feet}}{12 \text{ inches}} = 6.08 \text{ feet} \approx 6.1 \text{ feet}$$

Only 2 significant figures! How do I know? Hang around for 10 minutes and I'll tell you.

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The Path

If I didn't know the 1-step path, I need to find a longer path, but each step along the way is identical. I eliminate ONE unit and create a NEW UNIT.

$$= 73 \text{ inches } \frac{?\,mm}{?\ \text{inches}} \frac{?\,miles}{?\,mm} \frac{?\,yd}{?\,miles} \frac{?\,feet}{?\,yd} = ?\,\text{feet}$$

It's really just a whole series of multiplications by 1!

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Too Simple?

- As simple as that seems, the problems don't get any more difficult! There is more than 1 step, many different conversion factors, but the steps in solving the problem remain the same.

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Dimensional Analysis

1. Ask yourself what you know – with UNITS!
2. Ask yourself what you need to know – with UNITS!
3. Analyze the UNITS! change required.
4. Consider all the conversion factors you know (or have available) involving those UNITS!
5. Map the path.
6. Insert the conversion factors.
7. Run the numbers.
8. Celebrate victory!

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Another Example

- If there are 32 mg/mL of lead in a waste water sample, how many pound/gallons is this?

Do we recognize all the units?

mg = 10^{-3} g
mL = 10^{-3} Liters

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Another Example

- If there are 32 mg/mL of lead in a waste water sample, how many pound/gallons is this?

How would we solve this problem? What's the first thing to do?

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Dimensional Analysis

What do you know?

$$\frac{32\ mg\ lead}{mL\ water}$$

What do you want to know?

$$\frac{lb\ lead}{gal\ water}$$

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Dimensional Analysis

The path?

$$\frac{32\ mg\ lead}{mL\ water}????=\frac{?\ lb\ lead}{gal\ water}$$

Do you know a single step path?
Probably not, but what do we know?

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Dimensional Analysis

$$\frac{32\ mg\ lead}{mL\ water}????=\frac{?\ lb\ lead}{gal\ water}$$

mg measures mass of lead, lb measures weight of lead (same thing at sea level)

mL measures volume of water, gal measures volume of water

It makes sense that identical types of quantities are most easily converted into each other.

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Two Step Path

$$\frac{32\ mg\ lead}{mL\ water} \times \frac{?\ lb}{mg} \times \frac{?\ mL}{gal} = \frac{?\ lb\ lead}{gal\ water}$$

Do I know those 2 “single steps”?

Maybe I do, maybe I don’t. If I do, I can plug them right in. If not, I need to break them down into more steps.

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One possible path

$$\frac{32\ mg\ lead}{mL\ water} \times \frac{1000\ mL}{1\ L} \times \frac{1\ L}{1.057\ qt} \times \frac{4\ qt}{1\ ga} = \frac{121097\ mg\ lead}{gal\ water}$$

I’ve got the right units for water. Now, I need the right units for lead.

$$\frac{121097\ mg\ lead}{gal\ water} \times \frac{1\ g}{1000\ mg} \times \frac{1\ lb}{453.6\ g} = \frac{0.26697\ lb}{gal}$$

How should this number be expressed?

It SHOULD be written as 0.27 lb/gal, because only those two digits are significant. To write it as 0.26697 lb/gal implies that you know this number to 1 part in 100,000 rather than the 1 part in 100 that you really know.

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Sig Figs in a Calculated Answer

- Significant Figures represent the accuracy of a measurement – what if the answer isn’t measured but calculated?
- The calculated value must come from know values. These known values have accuracy of their own. Accuracy = sig figs
- You can determine the accuracy (sig figs) of a calculated value based on the accuracy of the values used to do the calculation.

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Calculating Sig Figs

2 different rules exist:

Multiplication/Division - the answer has the same number of sig figs as the digit with the least number of sig figs

Ex. 1.0 x 12.005 = 12

Addition/Subtraction - the answer has the same last decimal place as all digits have in common

Ex 1.1 + 2.222 + 13.333 = 16.7 (16.655 rounded)

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Helpful Hints

- When adding numbers in scientific notation, be sure the decimal points are in the proper place
- You can only add numbers that have the SAME UNITS!

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Sample Problem

$$6.24 \times 10^{-3} * 1.2406 \times 10^{4} * 6 = 464.48064$$
$$\approx 5 \times 10^{2}$$

- only 1 sig fig because of the “6”
- I write the answer in scientific notation so I don’t need zeroes as place markers (500)

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Sample Problem

$1.27 \times 10^2 + 1.6 \times 10^3 + 6.579 \times 10^5 = 659627$

How many sig figs? I don't know!
Line 'em up relative to the decimal point:

			1	2	7.
		1	6	?	?.
6	5	7	9	?	?.
6	5	9	6	2-ish	7-ish

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Sample Problem

$(6.24 \times 10^{-3} * 1.2406 \times 10^4) + 1.27 \times 10^2 =$

This problem involves both addition & multiplication!?!?!?

Simply apply each rule separately (obeying normal orders of operation) - BUT DON'T ROUND UNTIL THE END or you will introduce rounding errors.

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$(6.24 \times 10^{-3} * 1.2406 \times 10^4) + 1.27 \times 10^2 =$

First the multiplication (order of operations):
$6.24 \times 10^{-3} * 1.2406 \times 10^4 = 7.741344 \times 10^1$
But it's only 3 sig figs (multiplication rule):
$(7.74_{1344} \times 10^1) + 1.27 \times 10^2 = 204.41$

How many sig figs? Line 'em up!

	7	7.	4	1	3
1	2	7.	?	?	?
2	0	4.	4ish	1ish	3ish

So the "ones' place" is the last significant position.

So I write it as "204" or "2.04×10^2".

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Units and Math

You can multiply together any two numbers you want:

My height is 73 inches, my weight is 100 kg

73 inches * 100 kg = 7.3×10^3 kg-inches

When you multiply, the units combine.

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Addition/Subtraction and Units

You CAN'T add any two numbers, because the units don't mix:

73 inches + 100 kg = 173 ????

To add two numbers, they MUST have the same units!

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I have 48 cents in my pocket and $32 in my wallet. How much money do I have.

I can't just add them together:
48 cents + 32 dollars = 80 ???

But I can if I give them the same units:

$$48 \text{ cents} * \frac{1 \text{ dollar}}{100 \text{ cents}} = 0.48 \text{ dollars}$$

32 dollars + 0.48 dollars = 32.48 dollars (or $32.48)

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Same type – different units

All of the conversions we've done so far have been simply changing the unit of measure without changing the type of measurement.

Inches to feet. Inches measures length. Feet measures length.
mL to L to quarts to gallons. All measure volume.

That's great but kind of boring. I mean, I don't get any taller if I use inches instead of feet. I don't know anything new.

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Different unit. Different type.

OOOOO….NOW WE'RE TALKING!

Here's some real chemistry. If I can change a unit of mass into a unit of length, I've learned something new! But I need to have some physical relationship between length and mass for that to work.

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The most important chemical conversion!

$$poop\frac{???}{???} = gold$$

Sadly, there's no known relationship between poop and gold…at least not yet!

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What is Density?

Density is the mass to volume ratio of a substance.

It allows you to compare the relative "heaviness" of two materials. A larger density material means that a sample of the same size (volume) will weigh more.

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Ratios are Conversion Factors

Density is the ratio of mass to volume.

So, if you want to convert mass to volume or volume to mass – it's the DENSITY!

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$$Density\ of\ steel = \frac{3\ g\ steel}{mL\ steel}$$

It means 1 mL of steel has a mass of 3 g:

1 mL steel = 3 g steel

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Equalities are ratios – Conversion factors

1 mL steel = 3 g steel

$$\frac{1\ mL\ steel}{3\ g\ steel} = 1$$

$$\frac{3\ g\ steel}{1\ mL\ steel} = 1$$

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Conversion Factors

Powers of 1

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Sample problem

The density of aluminum is 2.7 g/mL. If I have a block of aluminum that is 1 meter on each side, then what is the mass of the block?

Where do we start?
We know the volume (length*width*height):
1 m x 1 m x 1 m = 1 m^3

Where do we want to go?
Grams (or kilograms or cg or some unit of mass!)

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Algebraically…

D = $\frac{\text{mass}}{\text{Volume}}$

But this is really just another conversion factor!

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1 m^3 * ???? = ? g

How do we go from m^3 to g?

m^3 is volume. g is mass. As soon as both are involved, there's a density somewhere!

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$$1\ m^3 \frac{2.7\ g}{mL} = ?\ g$$

That won't quite work – we need to get m^3 to mL so the mL will cancel.

Turns out 1 mL=1 cm^3. And 100 cm=1 m, so…

$$1\ m^3 = 1\ m \times m \times m = \frac{100\ cm}{1\ m}\frac{100\ cm}{1\ m}\frac{100\ cm}{1m}\frac{1\ mL}{1\ cm^3}\frac{2.7\ g}{1\ mL}$$
$$= 2.7 \times 10^6 g\ Al$$

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